Title
Multilevel sequential Monte Carlo samplers

Short abstract
This talk will review the probabilistic formulation of the inverse problem, the sequential Monte Carlo (SMC) sampling framework, and the standard multilevel Monte Carlo (MLMC) framework. These ideas will coalesce into the MLSMC sampling algorithm for Bayesian inverse problems. A numerical example of permeability inversion through an elliptic PDE given observations of pressure will illustrate the theoretical results.

Long abstract
Partial differential equations (PDEs) modeling physical phenomena are often defined only up to some unknown parameters, which may be high-dimensional or even function-valued. Given some observed data, one would like to invert for those parameters. It is natural to formulate the inverse problem probabilistically. In this case a prior probability distribution on the parameter of interest is updated to a posterior distribution conditioned on the observed data. The expected value is unique and it can be shown that the posterior distribution depends continuously on the data. Due to the cost of solving the PDE and the high-dimensional space, or in principle function-space, over which the probability distribution is defined, this is a computationally challenging problem.

Sequential Monte Carlo (SMC) samplers are a popular and versatile class of algorithms for such computationally intensive inference problems. One constructs a sequence of intermediate distributions and associated Markov chain kernels such that sequential importance sampling and resampling can guide the samples to the regions of high probability, hence avoiding degeneracy of the weights and explosion of the variance. For problems which admit a hierarchy of approximation levels, the multilevel Monte Carlo (MLMC) sampling framework enables a reduction of cost by leveraging a telescopic sum of increment estimators with vanishing variance.

This talk will review the probabilistic formulation of the inverse problem, the SMC framework, and the standard MLMC framework. These ideas will then coalesce into the MLSMC sampling algorithm for Bayesian inverse problems. The theoretical results will be illustrated by a numerical example of permeability inversion through an elliptic PDE given observations of the pressure.

Bio
Kody Law is a staff Mathematician in the Division of Computer Science and Mathematics at Oak Ridge National Laboratory. He received his PhD in Mathematics from the University of Massachusetts in 2010, and subsequently held positions as a postdoc at the University of Warwick and a research scientist at King Abdullah University of Science and Technology. He has published in the areas of computational applied mathematics, physics, and dynamical systems. His current research interests are focused on inverse uncertainty quantification: data assimilation, filtering, and Bayesian inverse problems.